DOCUMENT RESUME

ED 100 704 SE 018 621

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TITLE Applications of Piagetian Theory to the Teaching of Elementary Number Concepts: A Theory-into-Practice Approach.
PUB DATE [74]
NOTE 12p.; Best copy available

EDRS PRICE MF-$0.75 HC-$1.50 PLUS POSTAGE
DESCRIPTORS *Cognitive Development; Diagnostic Teaching; *Elementary School Mathematics; *Instruction; Learning; Learning Readiness; *Learning Theories; Mathematics Education; *Number Concepts
IDENTIFIERS *Piaget (Jean)

ABSTRACT

After a brief description of three Piagetian stages (pre-operational, transition to concrete operations, and concrete operations), the author identifies 12 key statements from Piaget's writings. These statements, together with related implications for teaching elementary school mathematics, methods of diagnosing stage acquisition, and examples are presented in chart form. (SD)

Applications of Piagetian Theory to the Teaching of Elementary Number Concepts: A Theory-into-practice Approach

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Introduction

Piaget has published three books which directly relate to the learning of mathematics:

1. Jean Piaget. The Child's Conception of Number. W. W. Norton & Co., Inc. New York, 1965.

2. Jean Piaget and Barbel Inhelder. The Child's Conception of Space. W. W. Norton & Co., Inc. New York, 1967.

3. Jean Piaget, Barbel Inhelder, and Alina Szeminska. The Child's Conception of Geometry. Basic Books, Inc. New York, 1960.

Each of these books employs the same style to lead the reader through the development of Piaget's theory and to convince the reader of the correctness of the proposed hypotheses. Piaget's style begins with his presentation of a problem (e.g., does conservation of number precede any numerical or quantifying activities by a child?), an experiment (or task) designed to answer the problem, results of the experiment (scripts of interviews with children as they work on the task), and finally, conclusions that can be drawn from the collective results. Piaget's style endeavors to follow the scientific method.

Although Piaget rarely applies his theory to teaching, such an application would be helpful and may, indeed, begin to provide solutions to the difficulties of teaching and learning mathematics. The following pages will show how Piagetian theory can be applied to the teaching and learning of some number concepts in grades K through 3 (or possibly later). All quotations are taken from The Child's Conception of Number.

A general Piagetian hypothesis is that logical thought is constructed by the individual ("logic is a construct", p. 204). Logical thinking cannot be taught; it can only be recognized and facilitated. Given an appropriate environment with which to interact, logical thought will be learned, i.e., constructed, in spite of another person (who is traditionally called "teacher").

Therefore, in a Piagetian spirit, the primary role of "teacher" is to recognize and facilitate the construction of logical thought, i.e., to provide an appropriate environment with which the individual learner can interact.

According to Piaget, the foundations of the concept of number (and the logic of classes) are constructed during ages 4 through 7½ for most children. In general, the full development of number concepts (logic of classes, one-one correspondence, additive and multiplicative compositions, and concomitant reversibility of thought) do not occur until the 8th year. It is useless and destructive to try to hasten this development.

Finally, Piaget theorizes that the symbolizing process is only a facilitator for the underlying logical or operational process. The forced use of symbols before the development of the underlying logical or operational processes would be artificial and destructive of growth.

The following pages provide a list of 12 hypotheses selected from Piagetian theory. Implications of each of these hypotheses are offered by the writer, as well as diagnoses and examples derived from Piaget's experiments. Hypotheses, implications for teaching, diagnoses, and examples are juxtaposed in four columns in order to show how theory can become practice.

Explanation of stages

Piaget sorts the results of the tasks/experiments described in The Child's Conception of Number into three stages. There are many different kinds of tasks/experiments, but children's responses to these can all be categorized according to the same criteria:

Stage I: These children are not at all successful with the tasks, not because they don't understand the directions (as some have claimed), but because their judgments are based on intuitions and perceptions. E.g., although a child has counted (or matched, one-one) 10 counters in one row and 10 counters in another row, he insists that the rows do not contain the same number of counters because one row is longer than the other. Thus, perception (length of row) wins out over logical compensation (space between counters). Another example: in one of Bil Keane's Family Circus cartoons, Jeffy says, as he faces a serving of squash, "Why did you cut my squash in half, Mommy? Now I have TWICE as much to eat." Jeffy is at Stage I--a piece of squash cut in two makes more to eat than the same piece in a single chunk.

Stage II: These children are at first unsuccessful, but later, with the experimenter's questioning, are able to begin judging logically or operationally. However, these logical judgments are short-lived because perceptions and intuitions continue to influence; there is a battle between perception and logic, with perception winning. E.g., although a child can make a one-one correspondence, he does not believe in the lasting equivalence of the corresponding sets.

Stage III: These children have completed the transition from intuitive and perceptual judgments to operational and logical judgments. Their judgments are correct, and they know this immediately, because their ability to reverse the operations mentally enables them to over-rule perceptual "distractions". E.g., a child asserts that there are 10 items in each of three sets, X, Y, and Z, even though he has counted the items in only one set, say, X. He is sure of himself because he has matched the items of X to the items of each Y and Z, and he believes that the matching is maintained, no matter how the items of each of the three sets are spatially located.

The teacher who would utilize Piagetian theory must be able to discriminate among these three stages of development for the given tasks. Only Stage III children would be diagnosed as conservers. Further elaboration of diagnoses and stages are given by Piaget in The Child's Conception of Number, with an explicit summary description on pp. 222-223.

HYPOTHESIS	IMPLICATIONS FOR TEACHING
1. Conversation with the child is much more reliable and more fruitful when it is related to experiments made with adequate material, and when the child, instead of thinking in the void, is talking about actions he has just performed. (p. vii)	Testing should be done one-one with objects rather than with paper-and-pencil. Teaching should include the manipulation of objects by both teacher and learner.
2. The construction of number goes hand-in-hand with the development of logic, and a pre-numerical period corresponds to the pre-logical level. (p. viii)	A child at the pre-logical level should not be asked to perform with numbers as if he were at the logical level.
3. Arithmetical notions acquire their structure because of conservation. (p. 4)	Make sure the child has the necessary conservations before expecting him to construct mathematical principles for himself.
4. Multiplication of relations makes the discovery of conservation of continuous quantity possible. (I.e., multiplication of relations is necessary but not sufficient for conservation of continuous quantity.) (p. 19)	Provide opportunities for experience in multiplication of relations, i.e., seriation of relations from two or more points of view simultaneously. E.g., comparing two quantities of lemonade from several points of view --height, cross-section, number of glasses, etc.--constitutes multiplication (two different quantities) of relations (how they are related).
5. For nonconservers of discontinuous quantity, the notion of quantity depends less on the verbal use of number names or on 1-1 correspondence, than on the global appearance of a set, and in particular, on the space occupied by it. (p. 45) Nonconservers do not differentiate between the number of objects and the space they occupy. It is only when the 1-1 correspondence becomes operational that the child feels certain of his counting. (p. 74)	A child's ability to count correctly or to match objects 1-1 is not sufficient evidence that he can meaningfully use numbers.

DIAGNOSIS	EXAMPLE
1. Testing should include the use of objects.	See #7 below for a complete list of tasks and their sources.
2. The tasks listed in #7 below have been analyzed by Piaget; Stage I and Stage II children would be characterized as pre-logical, whatever the task.	See #7 below for a complete list of tasks and their sources.
3. The tasks listed in #7 below are conservation tasks. Only Stage III children would be diagnosed as conservers.	See #7 below for a complete list of tasks and their sources.
4. The child conserves continuous quantities.	The child can multiply relations when he knows that the quantity of liquid in a long, narrow container remains the same when it is poured into a short, wide container. (The quantity is conserved.) He knows that the column of liquid in the first container is both higher and narrower than that in the second container because height and diameter compensate one another.
5. A child must demonstrate his understanding that quantities remain equivalent although the space occupied changes. He judges equivalence on the basis of logic rather than perception.	Before conservation of number (i.e., success at any or all tasks listed in #7 below), allow free play with objects, provide experiences of 1-1 matching (e.g., a store where objects are exchanged for pennies or chips, 1 for 1), and encourage different kinds of sorting. Do not introduce number symbols or operations.

HYPOTHESIS	IMPLICATIONS FOR TEACHING
6. Number is the fusion of class and asymmetrical relation (seriation) into a single operational whole. Class, asymmetrical relation, and number are three complementary manifestations of the same operational construction applied either to equivalences, differences, or to both together. (Logically: W = B + B' and B = W - B') (p. 184)	A child cannot "construct" numbers (beyond an intuitive sense of 1, 2, 3, or 4) until he can simultaneously deal with class inclusion and seriation.
7. The operations underlying the concept of number require mobility for carrying out the operations, for combining and separating them, for constructing and reconstructing simultaneously. These difficulties of synthesis are solved by the child's detachment from his pre-logical centration on perception and his growth toward the logical dynamism of reversibility. (p. 181)	Activity is both external and internal.

DIAGNOSIS	EXAMPLE
6. A child should demonstrate that he can deal with class inclusion problems and seriation problems before he is expected to deal with symbols for numbers or operations (with their symbols) on numbers.	Class inclusion materials: wooden beads (W) of two colors, say, brown (B) and white (B') The child demonstrates his understanding that since the class of wooden beads includes both the brown and the white beads, the class of wooden beads is larger than either subclass. (Logically: $W = B + B'$ and $B = W - B'$)
7. External activity is manifested by free play and by the results (visible and audible) of various conservation tasks. Internal activity is manifested by periods of quiet concentration (which ought not to be interrupted) and by analysis of the results of various conservation tasks.	Free play: with objects that will later be used in conservation tasks Tasks and analyses: Conservation of continuous quantities (e.g., water, clay) -- Ch. I Conservation of discontinuous quantities (e.g., coins, counters) -- Ch. II Provoked correspondence and equivalence of corresponding sets (provoked, i.e., artificially imposed by external circumstances, as between items of heterogeneous sets, e.g., eggs and egg cups) -- Ch. III Spontaneous correspondence--cardinal value of sets (spontaneous, i.e., in situations where the child is compelled to find the correspondence of his own accord and to make what use of it he can; the sets of objects are homogeneous, as coins with coins) -- Ch. IV Seriation, qualitative similarity and ordinal correspondence -- Ch. V Ordination and cardination -- Ch. VI Additive composition of classes--relations between class and number -- Ch. VII Additive composition of numbers and arithmetical relations of part to whole -- Ch. VIII Co-ordination of relations of equivalence and multiplicative composition of numbers -- Ch. IX Additive and multiplicative composition of relations and equalization of differences -- Ch. X

HYPOTHESIS	IMPLICATIONS FOR TEACHING
8. The operation of addition comes into being when, on the one hand, the addends are united in a whole, and on the other, this whole is regarded as invariant irrespective of the distribution of its parts. (p. 189) Numerical addition and subtraction become operations only when they can be composed in the reversible construction which is the additive "group" of integers, apart from which there can be nothing but unstable intuition. (p. 195)	A child can be taught to repeat formulae such as 2 + 2 = 4; 2 + 3 = 5; 2 + 4 = 6; etc., but there is no true assimilation until the child is capable of seeing that six is a totality, containing two and four as parts, and of grouping the various possible combinations in additive compositions. (p. 190)
9. One-one correspondence is an implicit multiplication.	When the child has established the correspondence between several sets, he will sooner or later become aware of this multiplication and use it as an explicit multiplication. (p. 205)
10. As soon as the child has grasped the relation of equivalence (X = Y, and Y = Z), he is capable of composing two of these relations. (p. 207) The equivalence of three sets is not more difficult to grasp than the equivalence between two, or, in other words, composition of two relations (in this case, equality) is as easy as the construction of one of them. (p. 208)	The transitive relation of equality can be used as soon as a child can conserve 1-1 correspondence between two sets.
11. As soon as the "two-to-one" relationship is grasped, it becomes generalizable to three, four, and five. In the various correspondences, one-to-one, two-to-one, three-to-one, etc., the value of each new set is no longer regarded only as an addition, but as a multiplication, "1 x n", "2 x n", "3 x n", etc. (p. 219)	Multiplication can easily be learned as a <u>multiple correspondence</u>.

DIAGNOSIS	EXAMPLE
8. Find out whether the child is capable of understanding that a whole remains constant irrespective of the various additive compositions of its parts. (p. 185)	The child is told that he is to have four sweets mid-morning and four for mid-afternoon. The next day he is to have the same number, but since he will be less hungry in the morning than in the afternoon, he will eat only one sweet in the morning and all the others in the afternoon. Beans are put before the child (or m & m's) to illustrate each statement, 3 beans being removed from one set of 4 and added to the other set to represent the position on the second day. The child is then asked to compare the two lots, 4 + 4 and 1 + 7 and to say whether he will eat the same number of sweets on both days. (pp. 185-186)
9. Allow the child to demonstrate 1-1 correspondence between sets. His understanding of the correspondence is based, not on perceptual likenesses of configurations, but on his immediate and spontaneous conviction that the correspondence is maintained in spite of spatial (perceptual) locations.	Egg cups and eggs, vases and flowers, are matched 1-1 and the matching endures regardless of spatial locations. The child is convinced, e.g., that there are 10 in each set of three sets, even though he has counted only one set.
10. Same as #9	Same as # 9
11. When a child demonstrates that he can conserve two-one correspondences, he is ready to begin learning multiplication as multiple correspondence.	The child establishes 1-1 correspondences between sets of vases and flowers (of several colors), one set (color) of flowers with the set of vases at a time. Then the child is asked how many flowers of each color can be placed in each vase, and whether each vase will contain the same number of flowers.

HYPOTHESIS	IMPLICATIONS FOR TEACHING
12. Measure is impossible without conservation of the quantities to be measured because quantities that are not conserved cannot be composed. (p. 225)	A child may be expected to measure quantities (continuous or discontinuous) when he has demonstrated that the quantity is conserved during transformations, and when he can use a common measure or a unit that either he has spontaneously designed or that has been imposed by instruction.

	DIAGNOSIS	EXAMPLE
12.	When a child discovers the idea of a common measure and of a unit he is ready to begin measuring meaningfully.	Spontaneous numerical measurement: The child is given two or three quantities of liquid in two or three containers of different shapes, such that he cannot estimate their ratio by direct perception. He is asked to say whether one of the quantities is equal to, greater than, or less than, one or both of the others, and is given some empty containers which he can use at will for the solution of the problem. The particular concern is to discover whether the child is capable of constructing a definite unit. (p. 222)

Piaget's life-long interest has been in investigating how children learn and come to think logically; his research has concentrated on theory-building. Although knowledge of Piagetian theory would be useful for teachers who would like to implement Piaget's ideas in the classroom, the necessary study would require years. Since most teachers don't have the time for such study, this theory-into-practice approach has been prepared in the hope of making more accessible and practical Piaget's contribution to how children learn beginning number concepts.